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CHOICE OF ASSET LIVES

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CHOICE OF ASSET LIVES^{**}

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Comments welcome

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Inflation, Discrete Replacement, and The Choice of Asset Lives

1. Introduction and Summary

Over the last two decades the rise in the inflation rate has motivated numerous studies of the effect of inflation on security prices and on investment. These effects are important for at least two reasons. First, managers argue that high and variable inflation rates make investment decisions more difficult. Second, there is a popular notion that firms are unwilling to invest in long-lived capital when the inflation rate is high. In this paper we explore the effects of inflation induced changes in nominal interest rates on the firm's choice of asset lives. Specifically, we address three questions:

1. Do high inflation rates cause managers to adopt short term technologies?
2. Does uncertainty about inflation change a firm's investment decisions?
3. How can a tax policy be designed to minimize the effect of inflation on real decisions?

When a firm considers acquiring an asset to accomplish some task, it usually has the choice of several different technologies. We focus on the choice between alternative technologies which are identical except for their purchase prices and useful lives. We assume the firm takes product prices, inflation and interest rates as given and chooses the optimal asset life using capital budgeting techniques. Changes in nominal interest rates affect

the value of nominal depreciation tax shields. Increasing the nominal interest rate decreases the value of these tax shields and the magnitude of this effect differs across asset lives. The initial nominal purchase price for a short-lived asset is lower than it is for the long-lived asset and the short-lived asset generates more depreciation tax shields. When the short-lived asset expires, the firm buys another - an outflow which is postponed if the long-lived asset is initially purchased. If the rate of inflation is positive, the second short-lived asset has a higher purchase price than the first and more depreciation tax shields. The choice between using the long-lived or short-lived asset depends on the tradeoff between the lower initial purchase price and higher tax shields of the short asset and the timing of subsequent cash outflows for future asset replacements.

We explore the relation between nominal interest rates and the relative demand prices of short and long-lived assets in Section 2. We find, in answer to our first question, that inflation induced changes in nominal interest rates do not unambiguously favor short-lived assets over long-lived assets. If interest rates are initially low, an inflation induced shift in interest rates favors short-lived assets, whereas if interest rates are initially high, long-lived assets are favored. For a uniform term structure our numerical results indicate that, for most asset life comparisons of interest, the changeover occurs at an interest rates between 5 and 25%. This implies that the greatest impacts of inflation on asset prices occur at rates which approximate recent experience.

The actual relative prices of long and short-lived assets are determined by the supply and demand for different lived assets. Nevertheless, it is likely that changes in nominal interest rates will lead to changes in the composition of the capital stock as one asset becomes dominant. These

changes make future relative asset prices uncertain. Section 3 explores the impact of uncertainty on asset choices. We show that, ceteris paribus, relative price uncertainty increases the value of short-lived assets relative to long-lived assets. Thus, in answer to our second question, uncertainty about future inflation rates does change investment decisions: an increase in uncertainty will induce firms to pay more for short-term technologies.

The impact of inflation on investment decisions is, due to its effect on nominal depreciation tax shields. When assets are depreciated over their useful lives, the response of the value of depreciation tax shields to inflation differs across assets of varying lives. In Section 4 we show that these distortive effects of depreciation tax shields can be eliminated by equalizing the tax service lives of assets which are substitutes. The reduction in the categories of useful lives in the Economic Recovery Tax Act of 1981 thus lessens the potential for inflation induced distortions in investment decisions.

The literature on the effects of inflation on investment is extensive and diverse, reflecting widespread academic as well as practical concern with the topic. Briefly, works relevant to our study can be divided into three categories. The first category includes papers which consider the impact of inflation and taxes on savings and on the aggregate level of investment in the economy. Feldstein is the principal investigator in this line of research: his work, including papers co-authored by Flemming [1971], Green and Sheshinski [1978, 1979], Poterba [1980], Slemrod [1980], and Summers [1977, 1978, 1979] is a substantial, multi-faceted literature in itself. His conclusion, based on a decade of theoretical, empirical and policy oriented research, is that "the interaction of inflation and the existing tax rules has contributed substantially to the decline of business investment in the

United States" (Feldstein, 1980, p. 5). Briefly, inflation operates on nominal depreciation deductions, interest deductions and the nominal interest rate: the net effect is to raise the corporate tax rate and reduce the real rate of return to capital. Savers then have less incentive to save and the supply of new capital therefore decreases. Using this paradigm, Feldstein and others investigate the behavior of aggregate investment relative to estimates of the real rate of return, the rate of return net of the cost of capital, and the real cost of capital services (based on a Hall-Jorgenson production function). These tests of highly aggregate series support the view that savings and investment are quite sensitive to changes in after tax real net returns and thus inflation induced changes in net returns are potentially quite distortive of real investment decisions.

The second category of research concerns the impact of inflation on relative asset values: specifically these investigations attempt to relate inflation-induced wealth transfers to cross-sectional variations in common stock returns. Recent research into these effects has been conducted by Hong (1977), French, Ruback and Schwert (1982) and Summers (1982). Although these investigations have proceeded separately from those of Feldstein et al, both rest on common theoretical underpinnings. The empirical tests generally begin with the observation that firms are holders and issuers long-term debt and non-indexed depreciation tax shields; these claims' value is affected by nominal interest rates and therefore by unexpected changes in the level of inflation. However, firms differ in their net positions and exposure to nominal contracts; therefore, systematic cross-sectional correlations between balance sheet exposures and the response of common stock returns to nominal interest rates should be observable. This argument appears reasonable and furthermore is consistent with Feldstein's contention that investors are

sensitive and able to respond to changes in real returns generated by changes in the value of nominal contracts. It is therefore surprising that the most carefully constructed tests of the "nominal contracting hypothesis" find that the wealth effects caused by unexpected inflation are not an important factor in explaining stock prices.

The macroeconomic and finance literatures appear to be at odds. The former offers evidence of significant effects of inflation generated by nominal tax deductions and nominal interest rates. The latter, after careful and extensive tests, is unable to find a statistically significant relation between unexpected inflation and common stock returns that can be linked variations in firms holdings of nominal tax deductions or nominal debt. This apparent contradiction indicates that the ways in which inflationary impacts are transmitted and distributed across firms are still not well understood. A resolution of these conflicting views is only possible through further research into the microeconomic effect of inflation on firms' decisions and a tracing through of related effects on the values of producers and purchasers of capital goods.

The third category of research, to which this paper belongs, deals with microeconomic effects: it considers the impact of inflation on capital budgeting, and on the choice among alternate technologies. Here the finance and economics literatures overlap. In finance, Van Horne [1971] first alerted practitioners and academicians to the need to make consistent inflation assumptions in estimating cash flows and the cost of capital for capital projects. Further, (although this was not emphasized) Van Horne implicitly recognized the nominal character of depreciation tax deductions in his calculations.

Using standardized investments in a capital budgeting framework, Nelson

[1976] analyzes the interaction of inflation and nominal tax shields to determine the impact of inflation on investment, the choice of a capital-labor ratio, choices among mutually exclusive projects, choices among mutually exclusive technologies of differing durability and the choice of a replacement policy. He finds that inflation could change the investment decision of a firm in each of the five dimensions considered. Regarding the choice between technologies of different durability, Nelson concludes that "[n]et present value rankings...will depend on the rate of inflation. Typically, rankings will change in favor of projects with lower durability at higher rates of inflation." By a numerical example, Nelson shows that the impact of inflation¹ actually is not monotonic: his longer-term (infinite life) project is preferred at inflation rates below 7.14% and above 33.3%, while the shorter-term (one year) project is preferred at inflation rates between 7.14% and 33.3%.

Hamada [1979] considers inflation and taxation in a variety of contexts, the most significant (for our purposes) being (1) taxes and fully anticipated inflation and (2) taxes, unbiased anticipations and random ex post inflation. Hamada argues in the former case that "the economy will have shorter-lived assets and will be less physical capital intensive, everything else the same," because the use of historical cost depreciation for tax

¹In Nelsons framework, inflation is measured via the Fisher relationship:
$$1 + p = (1 + r)(1 + R)$$
where p is the rate of change of prices, r is the real interest rate and R the nominal interest rate. In common with most others, Nelson assumes inflation rates can be perfectly anticipated and does not consider uncertainty about inflation nor the possibility that anticipated and actual rates of inflation might be different.

purposes affects the allocation of resources across asset classes and lives. Our analysis (Section 2) confirms that taxes and depreciation introduced into a world with or without inflation will cause firms to prefer shorter-lived assets. However, if inflation is introduced in a world where taxes and depreciation deductions already exist, then the impact of the change is ambiguous: for some asset life comparisons, an increase in inflation will favor assets with longer lives, while in other cases, shorter lived assets will come to be preferred.

In considering the effect of uncertainty, Hamada argues that, under assumptions of unbiased anticipations and risk neutrality, windfall gains and losses should lead to wealth effects consistent with the nominal contracting hypothesis, but that otherwise "rational decisions previously made should be unaffected." Hamada does not explicitly consider whether rational decisions under uncertainty differ in any way from decisions in a world where future inflation rates are fully anticipated and certain. We consider (in Section 3) the impact of changing degrees of uncertainty and find that greater uncertainty creates options for users of assets, and ceteris paribus biases choice in favor of shorter-term technologies.

Finally, in the economics literature, Auerbach [1979] and Abel [1981] each have considered the impact of inflation and taxes on optimal capital durability. Their models extend the Hall-Jorgenson [1967] framework in which an aggregate consumer-producer-investor allocates resources in a steady state economy with a Cobb-Douglas production technology and an exponentially decaying capital stock. Using this framework Abel [1981] derives results that are consistent with Nelson's and with ours: he shows that depending on the initial nominal interest rate, an increase in the rate of inflation first decreases, then increases the capital decay rate δ optimally chosen by

the investor. (The decay rate is inversely proportional to the durability of the capital stock.)

Our approach differs from the models of Abel and his predecessors in that, instead of employing a steady state model where homogeneous capital exponentially decays and is continuously replaced, we focus on values of alternative replacement programs having different discrete lives. Discrete replacement, we believe, more closely parallels the context in which firm's investment decisions are actually made, and that this is an important consideration for modelling the microeconomic impacts of inflation on investment.¹ Furthermore, the dynamic programming formulation permits the direct analysis of general term structures² and of generalized depreciation schedules (including those actually mandated by the Tax Code.) Finally, our approach can be extended to consider uncertainty: in particular it provides insight as to what uncertainties are relevant to decisions. For example, for a simple choice between assets of different lives, an investor facing a reasonably broad capital market needs only to take account of uncertainty about relative costs of the technologies at future replacement dates. Uncertainty about interest rates or inflation rates over the lifetimes of the assets are not relevant because in principle and in fact, firms can hedge their tax shields by selling or borrowing against them at prices determined by current nominal interest rates.

¹See Feldstein-Rothschild (1974).

²Characterized by vectors of pure nominal discount bond prices.

2. The Firm's Decision Problem

Consider a firm that wants to purchase a machine to accomplish some task. Assume that two machines are available: machine S that has a useful life of M periods and machine L that has a useful life of N periods. The optimal life for each machine depends on the time pattern of maintenance costs, resale prices, productivity and real interest rates. We assume that the real (constant dollar) value of operating costs and thus the optimal life of each machine is unaffected by inflation. The choice between machines L and S is based on the net present value of using each machine. Since the purchase of the machine is an outflow, the firm picks the machine with the least negative net present value. The choice between the two machines is affected by inflation because depreciation creates tax deductions which are nominal assets.

To analyze the firm's decision problem we make the following assumptions:

- (A. 1) Depreciation is deductible for tax purposes during an asset's useful life and the tax rate on corporate income is τ ;
- (A. 2) Assets qualify for an investment tax credit of γ times the purchase price of the asset;
- (A. 3) The firm realizes the full value of all tax shields.

Also, we define K_S and K_L as the cost (purchase price) of machines S and L , respectively; d_{st} and d_{Lt} are the proportion of the asset's purchase price that can be depreciated in period t ; and $P(0,t)$ is the current price of a pure discount bond which pays one dollar t periods hence. We use pure discount bond prices to calculate the net present value of cashflows for two reasons. First, it allows us to deal with a variety of term structures instead of artificially restricting the analysis to a uniform term structure. Second, since the current prices of pure discount bonds can be inferred from current coupon bond prices, we can separate uncertainty about

future interest rates from the problem of choosing among different lived assets. Uncertainty about future interest rates is embodied in equilibrium pure discount bond prices and the firm can eliminate this uncertainty by borrowing against future tax shields. Therefore, interest rate uncertainty is irrelevant for the choice among different lived assets.

The net present value of buying the shorter lived machine is

$$V_{SO} = K_S[-1 + \gamma + \tau \sum_{t=1}^M d_{st} P(0,t)] + P(0,M)E[\text{Max}(V_{SM}, V_{LM})] \quad (1)$$

and the net present value of buying the longer lived machine is

$$V_{LO} = K_L[-1 + \gamma + \tau \sum_{t=1}^N d_{Lt} P(0,t)] + P(0,N)E[\text{Max}(V_{SN}, V_{LN})]. \quad (2)$$

The first terms on the right hand sides of (1) and (2) are the after-tax cost purchasing machines S and L, respectively. The after-tax cost of the machines is their actual purchase price, less the investment tax credits and the present value of the depreciation tax shields. At the expiration of a machines' life the firm is required to purchase another machine and again has the choice of buying machine S or machine L. The second terms on the righthand sides of (1) and (2) are the present value of the optimal choice between machines S and L in the future. V_{st} and V_{Lt} are the certainty equivalent present values of the replacement programs t periods hence. Including the value of the future replacement decision makes (1) and (2) recursive; thus, V_{SO} and V_{LO} are the present values of an infinite replacement chain starting with machine S and machine L, respectively.

Our initial focus is on the effect of changes in nominal interest rates on

the demand prices of the short and long lived assets. That is, we consider the relation between nominal interest rates and the relative purchase prices of the short and long lived machines that makes the firm indifferent between the two assets. The machines are, by assumption, technologically identical; there is no reason to prefer one machine to another on the basis of performance. To prevent one asset from dominating the other, their values in use must be equal at each point in time:

$$V_{St} - V_{Lt} = 0 \quad (3)$$

We explore the effect of inflation induced changes in nominal interest rates by examining the difference between the present values of purchasing machines S and L that prevent dominance and thus satisfy (3). To allow for proportional changes in the value of the replacement programs over time we define:

$$\phi_t = \frac{E[V_{St}]}{V_{S0}} = \frac{E[V_{Lt}]}{V_{L0}} \quad .$$

While there are a variety of sources of changes in the value of the replacement programs over time, we focus on inflation induced effects. Since V_{St} and V_{Lt} are nominal values, inflation will, in general, cause ϕ_t to exceed unity.

The definition of ϕ_t and the non-dominance assumption provides an expression for the present values of the replacement programs;

$$V_{S0} = K_S[-1 + \gamma + \tau \sum_{t=1}^M d_{St} P(0,t)] + P(0,M) \phi_M V_{S0} \quad (4)$$

$$V_{LO} = K_L [-1 + \gamma + \tau \sum_{t=1}^N d_{Lt} P(0,t)] + P(0,N) \phi_N V_{LO} \quad (5)$$

Subtracting (5) from (4) and rearranging terms provides an expression for the difference between V_{SO} and V_{LO} ;

$$V_{SO} - V_{LO} = \frac{K_S [-1 + \gamma + \tau \sum_{t=1}^M d_{St} P(0,t)]}{1 - P(0,M) \phi_M} - \frac{K_L [-1 + \gamma + \tau \sum_{t=1}^N d_{Lt} P(0,t)]}{1 - P(0,N) \phi_N} \quad (6)$$

Since our concern is the effect of inflation induced changes in nominal interest rates on the relative demand prices of machines S and L , we define E as the difference between the net cost of the sequence of machines S and L ,

$$E = K_S [1 - P(0,M) \phi_M]^{-1} - K_L [1 - P(0,N) \phi_N]^{-1} . \quad (7)$$

Note that if the value of the replacement stream remained constant over time, E would measure the present value of the differences in the sequence of purchase prices for the two machines. Our expression (7) uses the generalized discounting factor $[1 - P(0,t) \phi_t]$ to incorporate possible changes in the values of the replacement programs over time.

Substituting (7) into (6) and rearranging terms expresses the difference in the present values of the alternative replacement programs in terms of the present value of the net cost difference:

$$\begin{aligned} V_{SO} - V_{LO} = & -E[1 - \gamma] + \tau \sum_{t=1}^M P(0,t) (K_L A_N (d_{St} - d_{Lt}) + d_{St} E) \\ & - \tau K_L A_N \sum_{t=M+1}^N P(0,t) d_{Lt} \end{aligned} \quad (8)$$

where A_N equals the discount factor $[1 - P(0, N)\phi_N]^{-1}$. Equation (8) shows that the difference in the values of using machine S and L is identical to the present value of an "investment" having outflow $E[1 - \gamma]$ immediately, inflows of $\tau(K_L A_N(d_{St} - d_{Lt}) + d_{St}E)$ during periods 1 through M, and outflows of $\tau K_L A_N d_{Lt}$ in each of the remaining periods. These "cash flows" are flows for a constructed investment: the actual difference in cash flows of V_{LO} and V_{SO} exhibit a different pattern. However, the constructed investment always has the same present value as the actual difference in cash flows. The cash flow pattern of this quasi-investment (outflows, inflows, outflows) suggests that level of interest rates will have a changing effect on the relative values of short and long-lived assets. For example, if the term structure is flat, (8) has the potential for two internal rates of return. At interest rates between these two internal rates of return the value of the short-lived asset exceeds the value of the long-lived asset, whereas the long-lived asset is more valuable than the short-lived asset at other interest rates.

2.1 Nominal Interests and Relative Demand Prices

We define $E^*(\underline{P})$ as the value of E that equates the value in use of each of the machines for a given vector of pure discount bond prices, \underline{P} ,

$$E^*(\underline{P}) = \frac{K_L A_N \tau \left[\sum_{t=1}^M P(0, t)(d_{St} - d_{Lt}) - \sum_{t=M+1}^N P(0, t)d_{Lt} \right]}{1 - \gamma - \tau \sum_{t=1}^M P(0, t)d_{St}} \quad (9)$$

$E^*(\underline{P})$ depends on both the real interest rate and the anticipated inflation rate through its dependence on pure discount bond prices. Once $E^*(\underline{P})$ is established for a given term structure, the prices of K_L and K_S are

fixed relative to one another so that $V_{S0} - V_{L0} = 0$ for inflation and interest rates that are consistent with the bond prices. The pricing relation (9) is well-defined in the sense that given the price of one machine, and current pure discount bond prices, the demand price of the other machine is uniquely determined through equation (7). The ratio of the demand prices of machines S and L , $\frac{K_S}{K_L}$, for a vector of pure discount bond prices, \underline{P} , is found by equating (7) and (9) and rearranging terms;

$$\frac{K_S}{K_L} = \left[1 + \tau \frac{\sum_{t=1}^M P(0,t)(d_{St} - d_{Lt}) - \sum_{t=M+1}^N P(0,t)d_{Lt}}{1 - \gamma - \tau \sum_{t=1}^M P(0,t)d_{St}} \right] \frac{1 - P(0,M)\phi_M}{1 - P(0,N)\phi_N} \quad (10)$$

The timing of depreciation tax shields does not affect the difference between the net cost of the two machines, $E^*(\underline{P})$, if all current discount bond prices equal one dollar. This corresponds to nominal interest rates, anticipated inflation rates, and real interest rates of zero. In this case, the sum of the discounted depreciation factors equals unity for each asset and the bracketed term in the numerator of (9) equals zero, implying $E^*(\underline{P})$ equals zero.

When nominal interest rates are positive, the timing of depreciation tax shields affects their value. The difference in the net cost of machines S and L that prevents one asset from dominating the other is determined by substituting current discount bond prices into (9). As shown in the Appendix A, the resulting value of $E^*(\underline{P})$ is always positive. A positive value of $E^*(\underline{P})$ indicates that the present value of the purchase prices of the short lived asset must exceed the present value of the purchase prices of the long lived machine to prevent machine S from dominating machine L .

A positive value of $E^*(P)$ indicates that, relative to a world with no taxes, depreciation tax deductions generally subsidize shorter-term assets. The price of a short-term asset relative to a long-term asset, will be higher in a world with taxes and depreciation deductions than in a world of no taxes or where depreciation is not a tax deductible expense. To see this, we use K_L as the numeraire, and $(\frac{K_S}{K_L})^0$ as the relative price prevailing in a world of no taxes and define $(\frac{K_S}{K_L})^C$ as the price prevailing in a world with both taxes and tax deductible depreciation. As before, we assume that future inflation rates are fully anticipated and are reflected in current nominal bond prices. From (7) and (9), and using the fact that $E^*(P)$ is positive (at positive interest rates) we have

$$K_{SM}^0 - K_L A_N = E^0 = E^*(P) = 0 \quad (11a)$$

$$K_{SM}^C - K_L A_N = E^C = E^*(P) \geq 0 \quad (11b)$$

Using the inequality, and rearranging terms, we obtain:

$$\frac{K_S^0}{K_L} \leq \frac{K_S^C}{K_L} \quad (12)$$

Relation (12) is strict inequality for every admissible term structure except one: if the nominal interest rate is zero in every period, then investors will be unaffected by the timing of depreciation deductions and relative prices will be the same as in a taxless world.

2.2 The Impact of Unanticipated Changes of Future Nominal Interest Rates

The difference in purchase prices depends on the level and pattern of the term structure of interest rates. We examine the differential for different levels of nominal interest rates by analyzing the behavior of price ratio $\frac{K_S}{K_L}$ as the term structure of interest rates shifts vertically. The motive for examining a vertical shift in the term structure is two-fold. First, it is analytically convenient since a change in the term structure of interest rates can be described with a single parameter. Second, and most importantly, it approximates the effect of an unanticipated change in the expected inflation rate on the term structure of interest rates. The impact of unanticipated changes in expected inflation rates on nominal interest rates has been studied extensively. Generally, theoretical and empirical work suggest that an unanticipated increase in the expected inflation rate leads to an increase in nominal interest rates. The studies do, however, differ on the magnitude of the inflation induced change. We circumvent this issue by assuming that an unanticipated change in expected inflation rates results in an exponential change in all discount bond prices. That is, denoting $P'(0,t)$ as the price of a pure discount riskless bond after the change in inflationary expectations,

$$P'(0,t) = \delta^t P(0,t)$$

This implies a proportional change in the yields of the discount bonds.

To analyze the impact of a shift in the term structure on the relative prices of the machines, $\frac{K_S}{K_L}$, an assumption about the impact of the shift on the value of future machine replacements is required. We assume that the change in the value of future replacements is proportional to the change in

discount bond prices so that the product $P(0,t)\phi_t$ is invariant to the term structure shift. This assumption is consistent with the indexation of nominal machine prices to the inflation rate and the independence of real interest rates and inflation.

A shift in the term structure of interest rates results in a change in the relative demand prices of machine S and L. The term structure shift parameter, δ^t , multiplies the pure discount bond prices in (10) to determine the price ratio that prevents dominance.

$$\frac{K_S}{K_L} = \left[1 + \tau \frac{\sum_{t=1}^M \delta^t P(0,t)(d_{St} - d_{Lt}) - \sum_{t=M+1}^N \delta^t P(0,t)d_{Lt}}{1 - \gamma - \tau \sum_{t=1}^M \delta^t P(0,t)d_{St}} \right] \frac{1 - P(0,M)\phi_M}{1 - P(0,N)\phi_N} \quad (11)$$

We show in Appendix A that $\frac{K_S}{K_L}$ is a positive function of the term structure shift parameter with at most one peak.

While the shape and intercepts of $\frac{K_S}{K_L}$ depend on the initial term structure of interest rates, the function's general shape - particularly its single peak - provides insight about the effect of term structure shifts on relative prices. If interest rates are low, that is, if nominal bond prices decline slowly with maturity, then a proportionate increase in nominal interest rates (holding real interest rates constant) causes $\frac{K_S}{K_L}$ to increase. However at some point, this effect reverses such that further increases in nominal rates causes $\frac{K_S}{K_L}$ to decline. Alternatively, if interest rates are initially high, that is, if nominal bond prices decrease rapidly, then a proportionate increase in nominal rates (holding real rates constant) causes $\frac{K_S}{K_L}$ to fall implying that any inflation induced increase in nominal interest rates benefits the longer-lived asset.

2.3 An Example

To further demonstrate the potential effect of inflation on the relative demand prices of alternative machines, in Figure 1 we plot $\frac{K_S}{K_L}$ as a function of the shift parameter for a uniform term structure in which the constant one period nominal interest rate initially equals zero (i.e., all riskless pure discount bond prices equal \$1). For $\delta = 1$ (no shift), $\frac{K_S}{K_L}$ equals $\frac{M}{N}$; as δ decreases, corresponding to an inflation-induced increase in the nominal interest rate, $\frac{K_S}{K_L}$ rises, reaches a maximum and then declines, approaching $\frac{M}{N}$ as δ approaches zero (infinite inflation). When $\frac{K_S}{K_L}$ is rising, higher nominal rates favor shorter-lived assets in the sense purchasers are willing to pay relatively more for the shorter-lived asset. Conversely, after $\frac{K_S}{K_L}$ reaches its maximum, further increases in nominal interest rates reduce the relative demand price of the shorter-lived asset favoring the longer-lived asset. This indicates that the nominal interest rate which maximizes $\frac{K_S}{K_L}$ is a critical value since it represents the rate at which increases in nominal interest rates switch from favoring the shorter-lived assets to favoring longer-lived assets.

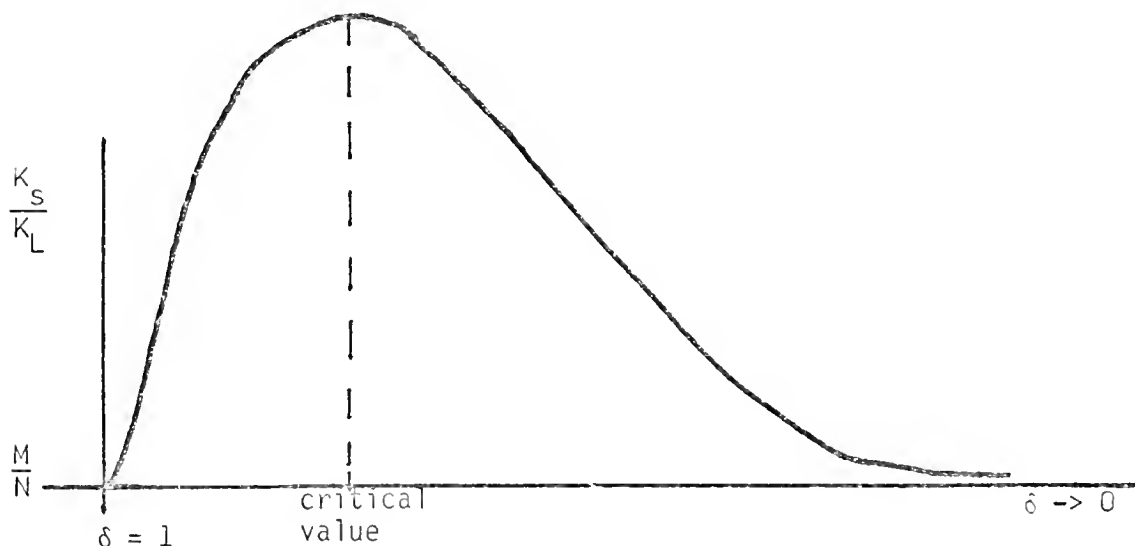
The nominal interest rate that maximizes $\frac{K_S}{K_L}$ depends on lives of the alternative assets. Table 1 displays critical values of the nominal rate for asset lives ranging from 2 to 50 years. The values in the table assume a flat term structure and are based on double-declining depreciation with an optimal shift to straight-line and no investment tax credits.¹ When nominal interest rates are below the critical value, an increase in nominal rates favors the short-lived asset and conversely. For example, if the nominal

¹Similar calculations using straight-line depreciation, yielded values 1-3 percentage points below those shown.

interest rate is currently 15% a small increase in the rate decreases the price of 50 year assets relative to all lives shown; decreases the price of 2-year assets relative to all but 50 year assets, etc. The ordering of relative prices changes follows the path of the 15% nominal interest rate isoquant represented as the dotted line in Table 1. The ordering is (50, 2, 40, 3, 30, 4, 5, 20, 15, 10) implying that an increase in nominal interest rates benefits the 50 year asset relative to the 2 (and all others), 2 relative to 40, 40 relative to 3, 3 relative to 30 etc. Further inspection of Table 1 indicates that at initial moderate nominal interest rates (10%-15%) unanticipated increases in nominal interest rates tend to benefit very long-lived (30-50 years) and very short-lived (3-5 years) assets relative to assets with lives in the 10 to 20 year range.

Figure 1

Demand prices of the short-lived asset relative to the long-lived asset $\left(\frac{K_S}{K_L}\right)$ as a function of the term structure shift parameter, δ .



Notes: This figure assumes a uniform term structure. Initially, all discount bond prices equal one dollar. For no shift in the term structure $\delta = 1$ and $\frac{K_S}{K_L} = \frac{M}{N}$ where M is the life of the short-lived asset and N is the life of the long-lived asset. As δ decreases, nominal interest rates increase and $\frac{K_S}{K_L}$ rises, reaches a maximum, and then declines approaching $\frac{M}{N}$ as δ approaches zero.

TABLE 1

Nominal Interest Rates Which Maximize the Price of the
Short-lived Asset Relative to the Long-lived Asset

<u>Life of the Long Asset</u>	<u>Life of the Short Asset</u>								
	2	3	4	5	10	15	20	30	40
3	64.3								
4	55.9	46.6							
5	50.5	42.2	37.1	(15%)					
10	35.1	29.4	25.8	23.3					
15	28.3	23.7	20.8	18.8		13.5			
20	24.2	20.3	17.9	16.1		11.7	9.5		
30	19.5	16.2	14.3	13.0	9.4	7.7	6.7		
40	16.7	14.0	12.2	11.1	8.1	6.7	5.8	4.8	
50	14.8	12.4	10.9	9.9	7.2	5.9	5.2	4.2	3.7
	15%								

Notes:

The relative price of the short-lived asset rises for a small increase in nominal interest rates if the rates are initially below those in the table and declines if the nominal interest rate is initially above those in the table. The calculations assume a uniform term structure, double-declining depreciation with an optimal shift to straight-line, no investment tax credit, and a real interest rate of zero. The dashed line is an isoquant of 15%.

An alternative way to represent the effects of changes in nominal rates is to calculate relative demand prices of different lived assets for a variety of nominal interest rates. The price ratio, $\frac{K_S}{K_L}$, is a positive single peaked function; it increases in the nominal interest rate until it reaches a maximum and declines thereafter. Table 2 presents $\frac{K_S}{K_L}$ for short asset lives of 3 to 20 years, long asset lives of 6 to 40 years, and nominal interest rates ranging from 0% to 40%, and indicates the magnitude of deviations in asset demand prices for different nominal interest rates. For example, for assets of relative lives of 3 and 6 years, a 5% nominal interest rate causes a 2 point (5%) rise in the value of the short asset relative to the long. Another 2-point rise occurs if the nominal interest rate reaches 15% and a subsequent 1-point rise as the nominal interest rate reaches 30-40%. Suppose a short-lived technology exists having a 5 point relative cost disadvantage (i.e., costing \$.55 to produce vs. \$1 for the long machine). For nominal interest rates between 0 and 20% the 6-year technology will be preferred to the 3 year technology. At nominal interest rates above 20% the preferred technology switches from 6 to 3 years. For very high nominal interest rates, a switch back to the 6 year technology occurs.

In contrast, low nominal interest rates have a greater impact on choices between longer lived technologies. Consider a choice between 20 and 40 year technologies with the same 5-point relative cost disadvantage for the short. When nominal interest are zero, the 40 year technology is preferred. However, when the nominal interest rate rises to 5%, the demand price of the short rises the 5 points required to overcome its relative cost disadvantage. The 20 year technology is preferred for nominal interest rates in the range of 5% to 10%, but between 10% and 15%, the relative price of the short falls so that for rates 15% and above, the 40 year technology is again preferred.

TABLE 2

Relative Demand Price for Machines of Various Lives
for Uniform Nominal Interest Rates¹

<u>Lives</u>		<u>Nominal Interest Rate</u>						
Long	Short	0%	5%	10%	15%	20%	30%	40%
6	3	.5	.521	.535	.543	.549	.553	.554
10	5	.5	.531	.546	.553	.556	.555	.551
20	10	.5	.547	.557	.556	.553	.544	.537
30	15	.5	.555	.557	.551	.545	.535	.528
40	20	.5	.558	.553	.545	.538	.528	.523

Notes:

¹The calculations assume double-declining depreciation with an optimal shift to straight-line, no investment tax credit, and a real interest rate of zero. The dashed lines are isoquants for relative asset prices equal to .55.

2.4 Equilibrium in the Asset Market

Our results indicate that, as nominal rates increase, the relative demand price of the short-lived asset rises, reaches a maximum and then declines. Actual machine prices are determined in the asset market as the demand and supply of assets equilibrate. The supply of different lived assets is determined by the costs of producing the assets and the market structure of the suppliers. If both the short and long-lived machines are produced in competitive constant cost industries, the relative supply prices of the machines are determined by the ratio of average production costs. Since we assume that machines S and L are identical in use, purchasers of the machines determine which asset to buy by substituting the supply prices into (1) and (2) and choosing the larger net present value. Equivalently, if the relative demand prices, $\frac{K_S}{K_L}$, from (10) exceed the relative supply prices, all purchasers will buy the short-lived asset. If the relative demand prices are less than the supply prices, the long-lived asset will be purchased.

The effect of the level of nominal interest rates on asset utilization can be inferred from Table 2. Assuming that machine L lasts twice as long as machine S and both are produced in competitive constant cost industries, the machine that will be purchased at different nominal interest rates is found by comparing the relative demand prices to the ratio of production costs. For example, if as before, the production cost of machine S is 55% of the production cost of machine L , the short-lived machine will be purchased for asset life and interest combinations within the dashed lines in Table 2 and the long-lived asset will be purchased otherwise.

Supply conditions other than competitive constant cost industries are, of course, feasible. One interesting alternative is when machines are produced in capital intensive industries with very low variable costs. With this

structure, both long and short-lived machines will co-exist at most interest rates. All of the wealth effects associated with changing relative demand prices will be borne by the asset suppliers. From the purchasers viewpoint, there would be no uncertainty about future relative asset prices. However, with the exception of this special case, inflation induced changes in nominal interest rates will lead suppliers to alter their production plans. Thus, uncertainty about future inflation rates creates uncertainty about future relative supply and about future relative prices. In the next section we show how this uncertainty changes relative demand prices in the present.

3. The Impact of Uncertainty

In this section, we examine the firm's optimal decision rule when future relative prices of assets are uncertain. We show that uncertainty about relative prices attaches an option value to the short-lived asset. This implies that the demand price of the short-lived asset (relative to the long) is higher under uncertainty than in the analysis in Section 2. If assets are priced at the relative demand prices established by (10), firms will optimally choose the short-lived asset. Thus, if general price inflation causes or is accompanied by higher price volatility¹ firms will tend to choose shorter-lived technologies as the inflation rate increases.

3.1 Breakeven Prices under Certainty and Uncertainty

The firm's actions are limited to the choice of the long- or short-lived asset. Therefore, its optimal policy can be characterized in terms of a

¹The model presented here draws upon and extends Baldwin and Meyer's (1979) analysis of an investor's optimal choice between unequal-lived opportunities.

breakeven difference between the cost of the long and short lived technology. In this section we make three simplifying assumptions. The first is that the cost of the long-lived asset is constant in real terms, whereas the cost of future short-lived assets is uncertain. Because the optimal decision rule involves only the difference in net costs, this assumption defines a numeraire and involves no loss of generality.

Second, we assume that the distribution of the short-lived asset's cost is time homogeneous in real terms. This implies that the investor's decision problem can be modeled as a stationary Markov decision process. Third, we assume a uniform term structure and define $\alpha^t \equiv \rho^t P(0,t)$, as the appropriate discount factor to apply to constant purchasing power flows received or paid t periods hence. The assumptions of time homogeneity and a uniform term structure simplify the analysis and clarify the economic issues behind our result. Our basic result - that price uncertainty biases choice in favor of short-run alternatives - is also obtained for a large class of Markov decision processes with non-uniform term structures and cost distributions which are changing over time.

In the model derived in Section 2, the investor is indifferent between a long and snort-lived asset only if $V_{SO} = V_{LO}$. Using Equation (6) (after substitution of α) the investor is indifferent if:

$$\frac{K_S^C}{K_L} = \frac{[-1 + \gamma + \tau \sum_{t=1}^N d_{Lt} \alpha^t]}{[-1 + \gamma + \tau \sum_{t=1}^M d_{St} \alpha^t]} \cdot \frac{[1 - \alpha^M]}{[1 - \alpha^N]} \quad (14)$$

The "C" superscript indicates that these are relative demand prices under future relative price certainty.

If future relative prices are uncertain, investors will generally be willing to pay a higher price for the short-lived asset. Appendix B shows that, under the assumptions given above, the breakeven demand price at which investors are indifferent between the short and long-lived alternatives is:

$$\frac{K_S^*}{K_L} = \frac{K_S^C}{K_L} + \frac{\alpha^M - \alpha^N}{1 - \alpha^N} \frac{1}{K_L} E \text{ Max } [K_S^* - \tilde{K}_S, 0] \quad (15)$$

The second term in (15) is positive implying that investors are willing to pay a premium to hold the short-term asset. The size of the premium is determined by a scale factor and an option factor. The scale factor adjusts for the unequal time spans committed to by decisions to purchase short or long-lived equipment and approaches zero as the assets' lives become equal.

The second factor indicates that the breakeven price premium on short-lived assets arises because an "option value" is attached to the shorter purchase cycle. The expectation, $E \text{ Max } [K_S^* - \tilde{K}_S, 0]$, is the value at expiration of a European put option exercisable at price K_S^* . In effect, if future prices are uncertain, each replacement gives the firm an option which is analogous to the right to put a security at price K_S^* . Replacements, with their implicit options occur more frequently for the short-lived asset, thus the short-lived asset's breakeven price increases relative to the long-lived asset. However, K_S^* , the "exercise price" controlling the put, is not exogenous as in standard options analysis, but is an endogenously determined by the firm's optimal decision rule.

The maximum in (15) is bounded below by zero, so the expectation is unambiguously positive. Also, the expectation is strictly increasing in the variance of \tilde{K}_S .¹ Therefore, an increase in uncertainty

¹The proof of the proposition is contained in Rothschild and Stiglitz (1970), Venezia and Brenner (1979) and Baldwin (1982).

(measured by investors' assessments of the dispersion of future relative prices around a mean) increases the option value of the short-lived asset. This increases the spread between the assets' breakeven relative prices under certainty $(\frac{K_S}{K_L})^c$ and under uncertainty $(\frac{K_S}{K_L})^*$.

It is plausible that economic forces related to inflation would also be associated with increases in relative price uncertainty. At a minimum, uncertainty about future inflation may be related to uncertainty about future nominal interest rates, making the relative value of future depreciation tax shields uncertain. As we showed in Section 2, for representative asset life comparisons, shifts in the nominal interest rate in the range of recent experience (0% to 20%) can cause fluctuations on the order of 10-15% in the relative net costs of alternate depreciable assets. This amount of variation in turn can increase the breakeven relative price by 1-2% over its certainty value. For example, let the price of the long asset be \$1.00, and suppose that under relative price certainty, the breakeven price of an asset lasting half as long is \$.50. If future relative prices are perceived to be uniformly distributed between \$.45 and \$.55, the breakeven price of the short asset becomes \$.5073, an increase of approximately 1.5%. Similar results hold for other distributional assumptions. Although the magnitude of this effect appears small it directly affects producers' revenues, and, with a fixed cost technology, has a correspondingly greater impact on producers' profits. Furthermore, depending on the producers' technologies and competitive market structure, in the short run, the shift in the breakeven price may be felt as a change in the quantity demanded rather than in the price.

To completely characterize the effect of an increase in relative price uncertainty is a complicated task outside the scope of this investigation. Any prediction of outcomes for an industry would require a detailed

specification of the supply side of the asset market, including information on producer technology, cost functions, adjustment costs, and the degree of producer diversification across substitute technologies. Adding to the general complexity are the problems that (1) assets are most likely to be imperfect substitutes in use and (2) the length of a replacement cycle is probably not fixed absolutely, but may itself be the result of an optimization decision.

Although incomplete, our analysis supports a conclusion that uncertainty about future relative prices gives a firm future production options it would not otherwise have. These options should be factored into decisions to purchase long or short-lived technologies, and can influence investment patterns. In general, uncertainty makes short-run technologies viable at higher realized prices than before: this effect is monotonic in the variance purchasers perceive to be associated with future relative prices. The short-run impact of inflation-induced uncertainty is to reduce current demand for longer-run technologies at previously-established relative prices. Intermediate and long run outcomes are more difficult to characterize, but it appears that changing degrees of uncertainty about future inflation could lead to significant repricing or else require costly adjustments in the types of assets demanded and supplied. Considerable turbulence in the markets for real assets, with commensurate impacts on market values of firms producing such assets may result from changing perceptions and uncertainty about inflation.

4. Tax Policy

The analysis in Sections 2 and 3 show that inflation induced changes in nominal interest rates affects the relative demand prices of different lived assets. These effects are due to the effect of inflation on the value of

nominal depreciation tax shields. In this section we show that a tax policy that equates the tax service lives of assets that are substitutes would eliminate these distortions.

In the context of the certainty analysis of Section 2, the price of a short-lived asset relative to a long-lived assets in the absence of depreciation tax shields is:

$$\left(\frac{K_S}{K_L}\right)^0 = \frac{1 - P(0, M)\phi_M}{1 - P(0, N)\phi_N} \quad (17)$$

In the examination of inflation induced shifts in nominal interest rates, we assumed that the change in the value of future replacement is proportional to the change in discount bond prices so that $P(0, t)\phi_t$ is invariant to term structure shifts. This assumption is consistent with the indexation of nominal machine prices to the inflation rate and the independence of real interest rates and inflation. Thus, in the absence of depreciation tax shields, inflation induced changes in nominal interest rates would not affect relative demand prices.

The changes in relative asset demand prices associated with inflation induced changes in nominal interest rates depend on tax policy. In our analysis of Section 2 we assume that depreciation is tax deductible over the assets' useful lives. This tax policy distorts relative demand prices since the changes in the value of depreciation tax shields resulting from an inflation induced change in nominal interest rates differs across assets of varying lives.

These distortive effects can, however, be eliminated by equalizing the tax service lives of assets which are substitutes. Suppose, for example, that machines S and L are substitutes and that both assets can be depreciated

over M periods. The value of using the short-lived machine is unchanged and is given by (4). The value of using the long-lived machine becomes:

$$V_{LO} = K_L [-1 + \gamma + \tau \sum_{t=1}^M d_{Lt} P(0,t)] + P(0, N) \phi_N V_{LO} \quad (18)$$

Since both assets are depreciated over the same time period the proportional depreciation terms, d_{St} and d_{Lt} , are equal. Using the techniques of Section 2 to determine the relative demand prices, $\frac{K_S}{K_L}$, we get

$$\frac{K_S}{K_L} = \frac{1 - P(0, M) \phi_M}{1 - P(0, N) \phi_N}$$

which is identical to the relative demand prices in the absence of depreciation tax shields. Thus, the inflation induced distortions in relative demand prices examined in Section 2 can be eliminated by equalizing the tax service lives of assets which are substitutes. Furthermore, since we assume that inflation induced changes in nominal interest rates do not affect the value of future replacements, relative asset prices are invariant to inflation and the effects of inflation induced uncertainty analyzed in Section 3 are also eliminated.

Recent changes in U.S tax policy have, to some extent, equalized the tax service lives of assets which are substitutes. The Economic Recovery Tax Act of 1981 equalizes the tax service lives within three broad categories: structures, equipment and light vehicles. Since assets which are substitutes are likely to fall within a given category, the distortive effects associated with an inflation induced change in nominal interest rates should be reduced.

5. Conclusion

This paper considers the impact of taxes and inflation on a firm's choice between assets of different lives. We model the firm's choice as a discrete replacement problem and investigate the firm's optimal policy and the behavior of relative prices under both certainty and uncertainty.

Under certainty, we find that at positive nominal interest rates, depreciation tax deductions bias choice against short-term assets. That is, the equilibrium price of short-lived assets relative to long will be higher when depreciation is a tax deductible, than in a world of no taxes or no depreciation deductions. However, assuming depreciation tax deductions are present, our analysis indicates that inflation induced changes in nominal interest rates can either increase or decrease the demand price of short-lived assets relative to long. The direction of the effect depends on the assets' relative lives and on the level of interest rates prior to the inflation induced change. Generally, if the initial level of the nominal interest rate is low, a small increase in the nominal rate raises the relative price of the short-lived asset; in contrast, if the initial level of nominal interest rates is high, a small increase raises the relative price of the long-lived asset. This changing effect of inflation on relative demand prices may interact with relative supply prices to alter the assets that are produced. We showed that given a constant cost technology, long-lived assets would be produced at very low and high interest rates whereas short-lived assets would be produced at intermediate levels of the nominal interest rate.

Our analysis shows that, through their effect on nominal interest rates, macroeconomic policies directed at other goals can incidentally change the relative attractiveness of depreciable assets with different tax lives. Because the effect is not monotonic the largest dislocations arise at moderate

inflation rates. Ironically, an inflationary shift in the nominal rate from 0 to 15% may cause more changes in relative prices and production plans than a shift from 0 to 100%. Furthermore, the effects of increases and decreases in nominal interest rates are symmetric. Once relative prices and production plans adjust to moderately high inflation and interest rates, disinflation, that is a decrease in inflationary expectations, will cause a reverse adjustment of relative prices, and lead to a reformulation of production plans, with associated wealth effects.

One way to reduce the distortions and dislocations caused by inflation induced changes in nominal interest rates is to assign equal tax lives to assets which are substitutes in use. The recent change in tax policy reduces the categories of useful lives, and thus, in addition to increasing tax subsidies to investment, the new rules reduce the potential for future distortions associated with changes in anticipated inflation and nominal interest rates.

Uncertainty about inflation may be related to both uncertainty about future nominal interest rates, and uncertainty about future relative prices of alternate technologies. However, for decision-making purposes all relevant information about future interest rates, and therefore the future value of depreciation tax shields is imbedded in the term structure of interest rates. Firms can therefore treat the value of depreciation tax shields as known quantities at the time an asset is purchased: operationally they can elect not to bear the risk of revaluation by hedging through borrowing or lease transactions.

Other sources of uncertainty affect future production and investment decisions, and thus cannot be hedged away. One source of "decisional uncertainty" that should be taken into account in analyzing investments is

uncertainty about future relative prices of alternative assets. We show that relative price uncertainty tends to bias choice in favor of short-term and against long-term technologies. Specifically, the breakeven price of a short-lived asset relative to a long-lived asset is higher under uncertainty than under certainty. This effect is monotonic in the variance of the perceived future price distribution. It arises because, if future prices are uncertain, a firm has an option to choose the cheaper technology at each replacement point. Replacement points are more frequent for short-term assets and thus their "option value" is commensurately higher.

The assignment of equal tax lives to assets which are substitutes in use makes relative prices invariant to inflation induced changes in nominal interest rates and the value of depreciation tax shields. This eliminates one potential source of uncertainty that can be directly linked to inflation. However, the effect of uncertainty from this source may be small relative to other environmental uncertainties indirectly linked to inflation. For example, uncertainty about the consistency of fiscal and monetary policy, about the permanence of the Tax Code and about factor prices may do far more to make firms' unwilling to commit to long-term investments than simple uncertainty about nominal interest rates operating on the value of depreciation tax shields.

Appendix A

Theorem. a. For real interest rates greater than or equal to zero and inflation greater than or equal to zero ($1 \geq \delta > 0$), $V_S - V_L$ is a single peaked function with at most two roots (zero-crossings).

b. E^* is a positive function of δ and has at most one peak.

c. $(\frac{K_S}{K_L})^c$ is a positive function of δ and has at most one peak.

Proof. a. We make use of the constructed investment of Equation (8). This investment is a polynomial in δ , the shift parameter, and has period "cash flows" (coefficients) as follows:

<u>Periods</u>	<u>"Cash Flows"</u>
0	$-E^*(1 - \gamma)$
1 - M	$\tau(K_L A_N (d_{St} - d_{Lt}) + d_{St} E^*) P(0, t)$
M - N	$-\tau K_L A_N d_{Lt} P(0, t)$

We begin by assuming a fact proved in (b), that $E^* > 0$ for pure discount bond prices less than one.

The proof of the theorem for straight-line depreciation is slightly different than for accelerated depreciation.

Straight-Line Depreciation. For straight-line depreciation, $d_{St} = 1/M$ and $d_{Lt} = 1/N$. Therefore $d_{St} - d_{Lt} > 0$. With positive E^* , this is sufficient for the signs of the coefficients to change only twice:

the pattern of signs is $(-, + + + \dots, - - - \dots)$. Invoking Descartes Law of Signs we have the function has at most two real roots on the interval $\delta = (1, 0)$. (All rates of inflation between zero and infinity correspond to points in that interval). Note that if $E^*(1 - \gamma)$, the coefficient of period 0 equals zero, the coefficients change signs only once, implying the existence of only one real root (at $\delta = 1$).

To show the function has a unique maximum: take the derivative of $V_S - V_L$ with respect to δ . The resulting function is also a polynomial; however, the zero-period term disappears. The new set of coefficients therefore has one sign change implying at most one real root, corresponding to a unique maximum.

Accelerated Depreciation. The simplest method of proof that we know uses the Pratt-Hammond extension of Descartes Law (1979): The number of roots is bounded by the number of sign changes in the partial sums of cash flows (coefficients of δ). For $E^* = 0$ and any declining balance, sum-of-the-years-digits or ACRS method of depreciation, the partial sums of the cash flows have the pattern $(0, + \dots +, 0)$ if $P(0, t)$ is constant for all t and $(0, + \dots +, +)$ if $P(0, t)$ declines with t . Note that the partial sums increase for positive coefficients ($1 \leq t \leq M$) and decrease when the coefficients turn negative ($t > M$).

Setting $E^* > 0$ results in negative $E^*(-1 + \gamma)$ being added to all partial sums, and smaller positive amounts being added to sums in period $1 - M$. The new sequence of partial sums must then have the pattern $(- \dots, + \dots, - \dots)$ or $(- \dots, + \dots)$. By the Pratt-Hammond rule, in the former case the number of roots is bounded by two, and there can be at most one maximum between the two roots. In the latter

case, there is at most one root, and the maximum occurs at $\delta = 1$ (no inflation).

b. To prove that E^* is greater than zero for positive inflation and positive real interest rates ($1 > P(0, t) > P(0, t + 1)$), we examine Equation (9) in the text. We assume that the investment tax credit γ , the tax rate τ and depreciation are set so that the total value of tax subsidies is less than the cost of the asset; i.e.:

$$1 > \gamma + \tau \sum_{t=1}^M d_{St} P(0, t)$$

In this case, the denominator of the expression is positive.

The sign of the numerator is determined by the sum:

$$\sum_{t=1}^N (d_{St} - d_{Lt}) P(0, t) ,$$

where $d_{St} \equiv 0$ for $t > M$. To show this quantity is positive, for all admissible term structures, define B as the first time period for which $d_{St} - d_{Lt} < 0$. Noting that $P(0, t) = P(0, B) + (P(0, t) - P(0, B))$ we have:

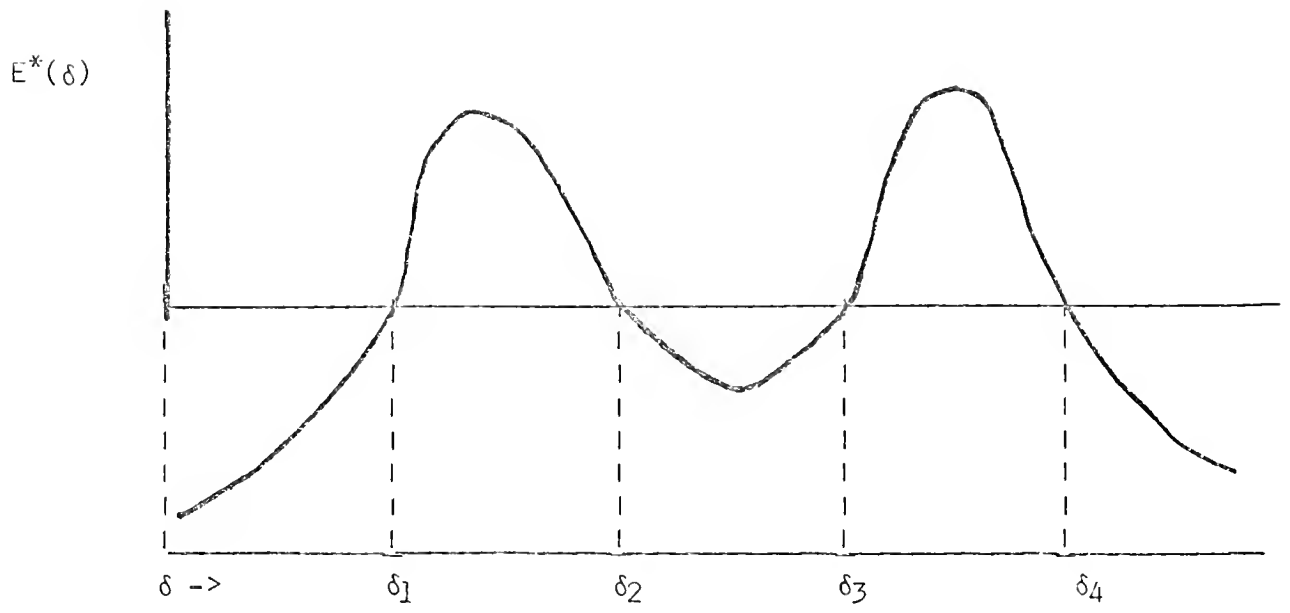
$$\begin{aligned} \sum_{t=1}^N (d_{St} - d_{Lt}) P(0, t) &= P(0, B) \sum_{t=1}^N (d_{St} - d_{Lt}) \\ &\quad + \sum_{t=1}^N (d_{St} - d_{Lt}) (P(0, t) - P(0, B)) \end{aligned}$$

Both depreciation schedules sum to 100% so the first term equals zero. In the

second term, for $t < B$, $d_{St} - d_{Lt} > 0$ by the definition of B , and $P(0, t) - P(0, B) \geq 0$ by the fact that discount bond prices are non-increasing. Conversely, for $t > B$, $d_{St} - d_{Lt} < 0$ and $P(0, t) - P(0, B) \leq 0$. Thus all elements of the second summation are positive.

To show that $E^*(\delta)$ has at most one peak, suppose, it has more than one (Figure A-1 depicts a function with two peaks.) By definition, $E^*(\delta)$ is constructed so that $V_S - V_L$ evaluated at E^* and δ is zero, i.e. each pair (E^*, δ) corresponds to a root of $V_S - V_L$. From the Figure, if E^* has two peaks, then some E^* would generate four roots $(\delta_1, \delta_2, \delta_3, \delta_4)$ of the function $V_S - V_L$. But (a) showed that $V_S - V_L$ (given E^*) has at most two roots. This contradicts the hypothesis and proves that E^* has at most one peak.

Figure A-1



c. To show $\frac{K_S}{K_L}$ is positive and single-peaked, recall

$$E = K_S A_M - K_L A_N$$

Substituting E^* for E and rearranging terms:

$$\left(\frac{K_S}{K_L}\right)^C = \frac{A_N}{A_M} (E^* + 1)$$

The assumption of neutrality implies A_N and A_M are independent of δ , thus $\left(\frac{K_S}{K_L}\right)^C$ is a linear transformation of E^* . $\left(\frac{K_S}{K_L}\right)^C$ is positive because A_N , A_M and E^* are positive. $\left(\frac{K_S}{K_L}\right)^C$ is single-peaked because E^* is single-peaked.

Appendix B

The Firm's Decision Problem Under Uncertainty

Recall that two assets, machine S and machine L are always available to the firm. If S is purchased the firm will buy a new machine (S or L) at the end of M period; if L is purchased a new machine is required N period hence. The possibility of changing the useful life of either machine is not considered. Define $v(t)$ as the value of the firm's capital program at time t assuming the firm follows an optimal policy from t onward. The dynamic program as of $t = 0$ can be written as:

$$v(0) = \text{Max}[V_{SO}, V_{LO}] = \text{Max} \{J_{SO} + P(0,M)E_v(M); J_{LO} + P(0,N)E_v(N)\} , \quad (\text{B-1})$$

where V_{SO} and V_{LO} are as defined in equation (1) and (2); and J_S and J_L are the net costs of choosing the short or long lived machine, respectively.

$$J_{SO} \equiv K_{SO}[-1 + \gamma + \tau \sum_{t=1}^M P(0, M)d_{St}] , \quad (\text{B-2a})$$

$$J_{LO} \equiv K_{LO}[-1 + \gamma + \tau \sum_{t=1}^N P(0, N)d_{Lt}] . \quad (\text{B-2b})$$

The optimal policy can be characterized as breakeven price difference between the long and short-lived technology:

$$[J_{SO} - J_{LO}]^* = P(0,M)E_v(M) - P(0,N)E_v(N) \quad (\text{B-3})$$

Under the optimal policy, if $J_{S0} - J_{L0}$ is greater than $(J_{S0} - J_{L0})^*$ (i.e. the short-lived asset is relatively less expensive), then the firm should purchase the short-lived asset; if $J_{S0} - J_{L0}$ is less than $(J_{S0} - J_{L0})^*$ the firm should purchase the long-lived asset.

Recall that, to simplify the analysis, we assumed

- 1) that J_{Lt} is constant in real terms, whereas J_{St} is uncertain;
- 2) that the distribution from which J_{St} is drawn is time homogeneous in real terms; and
- 3) that α^t is the discount factor applicable to constant purchasing power flows t periods hence.

Theorem. Let a "C" superscript denote the relative demand price if future relative prices are certain and a "*" superscript denote the breakeven demand price if future relative prices are uncertain.

a. Let uncertainty be attached to any of the factors affecting J_S (see B-2a). Then

$$\left(\frac{J_S}{J_L}\right)^* > \left(\frac{J_S}{J_L}\right)^C = \frac{1 - \alpha^M}{1 - \alpha^N} .$$

b. Let uncertainty be attached to K_{St} . Then

$$\left(\frac{K_S}{K_L}\right)^* = \left(\frac{K_S}{K_L}\right)^C + \frac{\alpha^M - \alpha^N}{1 - \alpha^N} \frac{1}{K_L} E \text{ Max } (K_S^* - \tilde{K}_S, 0) .$$

(Result (b.) is the result cited in the text. Result (a.) is more general in

that more sources of uncertainty are encompassed within the formulation.)

Proof. a. The equality follows from Equation (6) in the text and the definitions of α , J_S and J_L . (See also Equation (14), text.)

From Equation (B-2), for a time homogeneous Markov process, using the fact that J_L is non-random, optimal J_S^* satisfies:

$$\frac{J_S^* - J_L}{-\alpha^M + \alpha^N} = Ev \quad (B-4)$$

Define G as the probability that the firm will purchase a short-lived asset given whatever decision rule it applies. Define $\frac{R}{G}$ as the firm's expected net cost conditioned on its purchase of a short-lived asset under the decision rule.

From (B-1) and these definitions the expected value of the replacement program can be rewritten:

$$\begin{aligned} Ev &= G\left(\frac{R}{G} + \alpha^M Ev\right) \\ &+ (1 - G)(J_L + \alpha^N Ev) \end{aligned} \quad (B-5)$$

Rearranging terms, and applying the optimality condition (B-4) yields

$$\frac{R^* + (1 - G^*)J_L}{1 - \alpha^N - G^*(\alpha^M - \alpha^N)} = Ev = \frac{J_S^* - J_L}{-\alpha^M + \alpha^N} \quad (B-6)$$

where stars (*) denote that this relationship holds only for the optimal

breakeven price. (B-6) can be rewritten:

$$\frac{J_S^*}{J_L} = \frac{1 - \alpha^M}{1 - \alpha^N} - \frac{(\alpha^M - \alpha^N)}{1 - \alpha^N} \frac{1}{J_L} (R^* - G^* J_S^*) \quad (B-7)$$

\parallel
 $\left(\frac{J_S}{J_L}\right)^C$

In (B-7), J_L and J_S are negative and $R^* - G^* J_S^*$ is non-negative for any value of J_S^* .

Suppose J_S^* were set equal to $J_L \frac{(1 - \alpha^M)}{(1 - \alpha^N)}$. Equation (B-7) implies that this policy is not optimal unless $R - G J_S$, evaluated at this J_S^* is zero, i.e. unless the firm would never choose the short-term asset. For this rule to be optimal, the probability distribution of annuitized J_S must lie entirely below the annuitized cost of the long-lived asset.

In contrast, suppose there is a non-zero probability that

$$\frac{J_S}{1 - \alpha^M} > \frac{J_L}{1 - \alpha^N} .$$

That is, adjusted for relative durability, the short asset is sometimes cheaper than the long. The second term in (B-7) must then be positive implying

$$\left(\frac{J_S}{J_L}\right)^* > \left(\frac{J_S}{J_L}\right)^C . \quad \text{Q.E.D.}$$

b. Substituting for J_S and J_L in terms of their definitions (B-2a and B-2b), are arranging and cancelling terms as appropriate, yields

$$\left(\frac{K_S}{K_L}\right)^* = \left(\frac{K_S}{K_L}\right)^C - \frac{\alpha^M - \alpha^N}{1 - \alpha^N} \cdot \frac{1}{K_L} \int_0^{K_S^*} (K_S - K_S^*) dF(K_S) \quad (B-8)$$

(The integral in (B-8) is derived from appropriate transformations of integral equivalents of the right expectations R and GJ_S in (B-7).) It is straightforward to verify that

$$\begin{aligned} - \int_0^{K_S^*} (K_S - K_S^*) dF(K_S) &= -E \text{ Min } [\tilde{K}_S - K_S^*, 0] \\ &= E \text{ Max } [K_S^* - \tilde{K}_S, 0] \end{aligned} \quad (B-9)$$

Substitution from (B-9) into (B-8) proves the result.

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